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LETTER TO THE EDITOR

Noise-enhanced domain coarsening in ordering dynamics of lamellar patterns

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Abstract. The coarsening of domains of lamellar patterns in the presence of an additive noise is studied in numerical simulations of the cell-dynamical-system model of the Swift–Hohenberg equation. We demonstrate the noise-enhanced domain growth. The growth exponent of domain size depends on the noise intensity, which implies a breakdown of universality.

Coarsening phenomena are ubiquitous in nature. A typical example is the dynamics of phase separation in systems quenched from a high-temperature disordered state into an ordered two-phase region [1]. A general consensus of many theoretical and experimental studies on these systems is the scale-invariant morphology that develops at the late stages of the coarsening process; the structure at different times is statistically similar and the spatial distribution of domains is described by a single time-dependent length scale (the dynamic scaling hypothesis). At the same time the *universality hypothesis* has been generally adopted that details of the system should not affect universal quantities such as the growth exponent.

In this paper we extend this question to systems in which the ordered phase is a periodic lamellar state. Rayleigh–Bénard (RB) convection is a canonical example of such systems. When a horizontal layer of fluid is heated from below and driven far from equilibrium, it undergoes a transition from a spatially and temporally homogeneous conduction state to a convective state. The structure that emerges above the convective threshold in large aspect-ratio systems is the stripes (convective rolls) of arbitrary orientation. The subsequent evolution of the pattern involves the reorientation of rolls and elimination of defects to attain parallel rolls of sizable extent. Other examples include diblock copolymers and chemically reactive binary mixtures where competing interactions result in stable lamellar phases. Because of the existence of the spatial period $2\pi/q_0$ of the ordered structure, the dynamics of phase-ordering of the lamellar states is quite intriguing in comparison with the case of phase separation for which $q_0 = 0$, and has been under extensive investigation (mostly by numerical simulations) [2–7]. Unfortunately, however, no successful theoretical formulation is yet available for the problem. Thus disagreement has remained concerning the (intermediate) asymptotic growth law of the domain coarsening.

There is also the question of noise effects on pattern-forming nonequilibrium systems. For ($q_0 = 0$) phase separating systems, fluctuation of thermal origin seems asymptotically irrelevant in the scaling regime; temperature effects are controlled by a zero-temperature fixed point [8]. The main effect of thermal noise then is only to roughen the domain walls, and the approach to the asymptote is slower for stronger noise [9]. This is in accord with the intuitive image of noise we have developed from equilibrium thermodynamics that noise induces disorder. However, we should be aware of the fact that sometimes the notion we usually associate to internal noise (derived from the many microscopic degrees of freedom of the system) can be seriously misleading. In fact we have recently witnessed counterintuitive examples under certain circumstances. Archetypical examples are noise-induced phase transition [10] and stochastic resonance [11]. The stochastic force associated with such effects is known as external noise. Along different veins several noteworthy features within the context of pattern formation have also been suggested. Namely, with noise added, (i) convective rolls appear in RB systems in which a deterministic analysis predicts a homogeneous solution [12]; (ii) the characteristic size of spatial structure that emerges during a slow sweep of the bifurcation parameter through its threshold increases (logarithmically) with the magnitude of added noise [13]; (iii) it drives the system toward a preferred wave number of the patterned state which depends only on the system parameters [14].

Simulations [2–7] have given various results for the growth exponent (α) of the domain size of stripes, including $\alpha = 1/2$, $1/4$ and $1/5$. A general opinion in the recent literature is that the scaling exponents are $1/5$ and $1/4$ under zero noise and finite noise, respectively, leading to the possibility of a new universality class. The important question is then 'is the presence or absence of noise the main feature determining the universality class?'. However, there is no systematic study to answer this question. The present study deals with this issue.

We study the influence of noise on the coarsening kinetics of the lamellar patterns by numerical simulations. To be explicit we will treat a computationally efficient cell-dynamical-system (CDS) version [15] of the Swift–Hohenberg (SH) model [16] that was originally introduced to investigate the RB instability. (Although the SH model does not allow for precise prediction of the convection system since it neglects non-Boussinesq effects and the mean-flow effect as well, the model constitutes a paradigm of pattern formation outside of equilibrium, exhibiting many features common to natural patterns [17]. In the following text, however, we shall use hydrodynamic terms inherent in the convection system for convenience.) It is a two-dimensional theory involving a real order parameter which describes the slow (spatial and temporal) variation of the vertical component of the fluid velocity. It reads as

$$\psi(n, t + 1) = A \tanh \psi(n, t) - L[[\psi(n, t)]_c]_c + B\eta(n, t) \tag{1}$$

with $[\psi]_c \equiv \langle\langle\psi\rangle\rangle - c\psi$, where $\psi(n, t)$ is the order parameter in the nth 'cell' at time t. The positive constants A, L, c and B are parameters of our model, B being the noise amplitude. The noise field $\eta(n, t)$ is taken to be a random number, uniformly distributed in the interval $[-1, 1]$, assigned at each time t to each cell site n. The noise can have an internal (thermal) or external origin with respect to the system under study. (We have also performed simulations in which noise has a Gaussian white distribution. The result was quite insensitive to the choice.) The operator $\langle\langle\rangle\rangle$ is the isotropic spatial average [18], and defined on the square lattice by $\langle\langle\psi\rangle\rangle = (1/6)\sum\psi$(nearest neighbour cells) + $(1/12)\sum\psi$(next-nearest-neighbour cells). In this model the wave vector of the most unstable mode of the linear dispersion (with respect to the conducting state) is given by $q_m = \arccos[(3c - 1)/2]$. For more details on the CDS model (1) and its relation to the SH model, we refer the reader to [3] and [15].

We have performed the numerical simulation of equation (1) on a square lattice of 1024×1024 cells with periodic boundary conditions with parameters $A = 1.025$, $L = 0.8$, $c = 0.7$ at variable values of B ranging from 0 to 0.3; the aspect ratio Γ (system length versus $2\pi/q_0$) is therefore $\Gamma \sim 160$. The initial distribution of the ψ's is specified by a random uniform distribution in the range $[-0.01, 0.01]$. Each run is repeated with five different initial configurations to average over.

We computed the circularly-averaged scattering function $S(q, t)$ defined by $S(q, t) =$

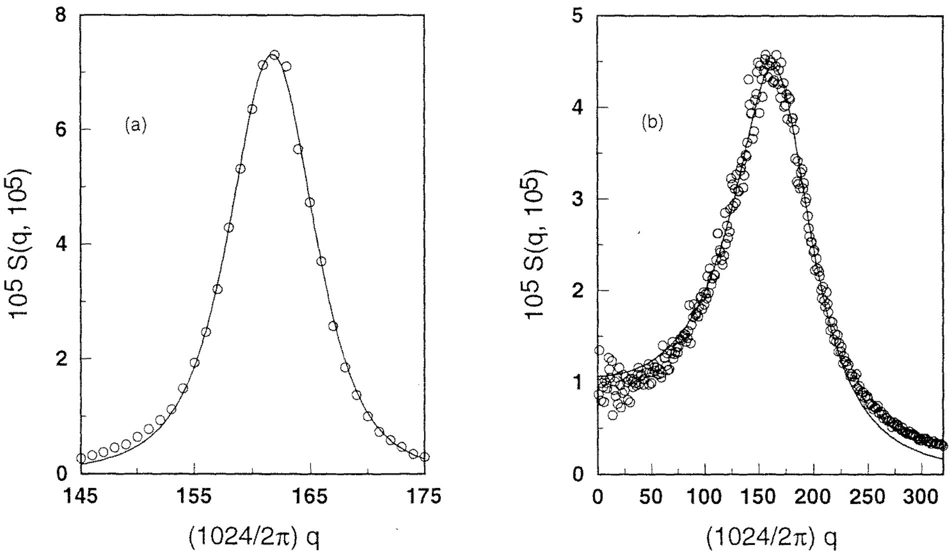

Figure 1. The circularly-averaged scattering function $S(q, t)$ at time step $t = 10^5$ as a function of the wave number q. In (a) and (b), $S(q, 10^5)$ is shown for the noise strength $B = 0.03$ and 0.2, corresponding to the ordered and disordered states, respectively. The q is defined only for discrete multiples of $2\pi/1024$, and $S(q, t)$ is in arbitrary units. Note the differences in axis scaling between the two plots. The data (open circles) were hardened using transformation $\psi \to \mathrm{sgn}\psi$ to remove any effect due to the finiteness of the ratio of the thickness of domain walls to the domain size. The solid curve is the best fit to the data using a squared Lorentzian (a) or a Lorentzian form (b).

$\langle \psi(\boldsymbol{q}, t)\psi^*(\boldsymbol{q}, t)\rangle$ with $\psi(\boldsymbol{q}, t)$ being the Fourier transform of the order parameter, and the orientation of the wave vector $\boldsymbol{q}$ was averaged over. Time evolution of $S(q, t)$ exhibits narrowing of the scattering profile and increase of the peak intensity at the position $q \approx q_m (= 0.99)$. In figures 1(a) and 1(b) the scattering functions $S(q, t)$ are shown for $B = 0.03$ and 0.2, respectively. We fitted $S(q, t)$ for $B \leqslant 0.07$ to a squared Lorentzian form [5, 6]

$$S(q, t) = a^2/[(q^2 - b)^2 + c^2]^2 \qquad (2)$$

and extracted the half width at half maximum $\ell^{-1}(t)$, and the peak height $S_p(t)$. (We have also performed a fit to a Gaussian form and this made no difference to our results.) For the stronger noise strength $B > 0.07$, the squared Lorentzian fit shows systematic deviation in the peak and tail regions, and a Lorentzian fit $S(q, t) = a/[(q^2 - b)^2 + c^2]$ was a significantly better fit, suggesting a disordered state in this parameter range (see below). The value of the peak position ($q = \sqrt{b}$) determined from the fitting could not be distinguished from q_m within numerical uncertainty. The length $\ell(t)$ measured in this way is displayed in figure 2. As seen from the figure, $\ell(t)$ at the late stage of coarsening is one order of magnitude smaller than the system length. Hence we believe that the finite size effects are negligible in our simulation. The characteristic length scales of the late stage are well fitted by a power law $\ell \propto t^\alpha$ over more than two decades in time. This same scaling was found for the peak height, $S_p(t) \propto t^\alpha$ (as seen in the inset to figure 2(a)), confirming the scaling form of $S(q, t)$ [2, 7, 6]: $S(q, t) = \ell(t) f((q - q_m)\ell(t))$, where $f(x)$ is a scaling function. There are two striking features to be noticed in figure 2. First, we find that at low noise strengths (figure 2(a)) adding noise speeds up the coarsening as revealed by the increasing slopes of the curve with B. This behavior is also reflected in the pattern as seen in figure 3, which depicts configurations at $t = 10^5$. One sees that a higher value of B (figure 3(b)) results in a larger average domain

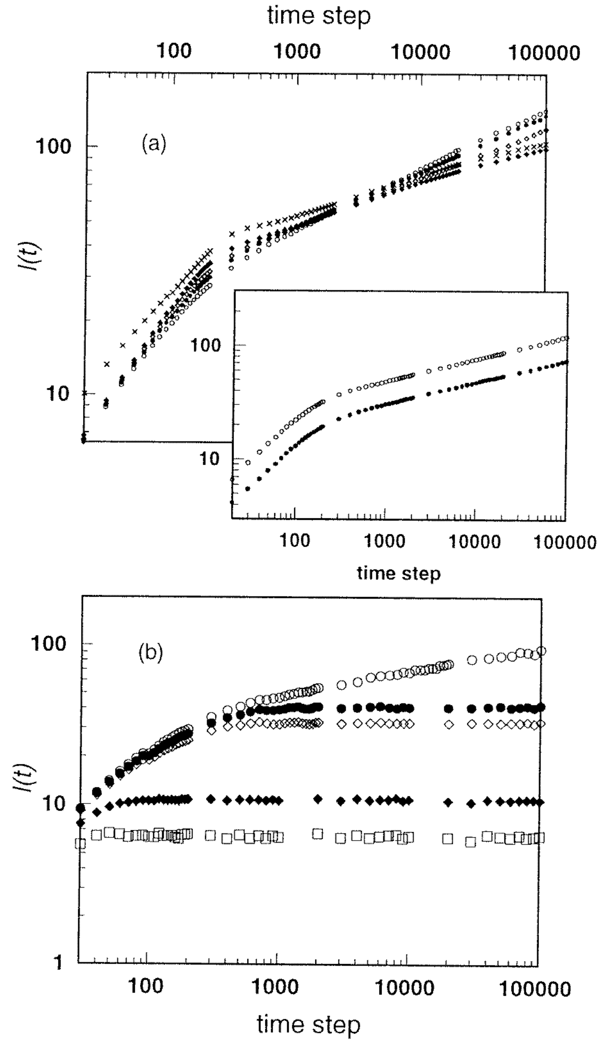

Figure 2. Time evolution of the half width at half maximum $\ell^{-1}(t)$ of the scattering function for different noise strengths; (a) $B = 0.07, 0.05, 0.03, 0, 0.01$, and (b) $B = 0.08, 0.09, 0.1, 0.2, 0.3$, from the right-hand top to bottom, respectively. In the inset to (a), time evolution of $\ell(t)$ (open circles) and the peak intensity $S_p(t)$ (filled circles) of the scattering function for $B = 0.03$ is shown.

size. Second, for the stronger noise a steady state was rapidly reached (figure 2(b)). The state achieved is characterized by the small range of spatial correlation, which is only of the order of the wavelength of the rolls ($\sim 2\pi/q_m$). The qualitative difference between the low (ordered) and high noise-strength (disordered) states is also apparent in the one-point distribution function $\rho(\psi)$ displayed in figure 4(a). As a matter of fact, Elder *et al* [2] have already found this disordered state in which ρ is single-peaked at $\psi = 0$, and called it the *isotropic phase*. The use of this term (drawn from analogy with liquid crystals) is somewhat misleading, however, since we find (figure 1(b)) the scattering function $S(q, t)$ for this state is still peaked at $q \approx q_m$.

The growth exponent α is plotted as a function of the noise strength B in figure 4(b). It is

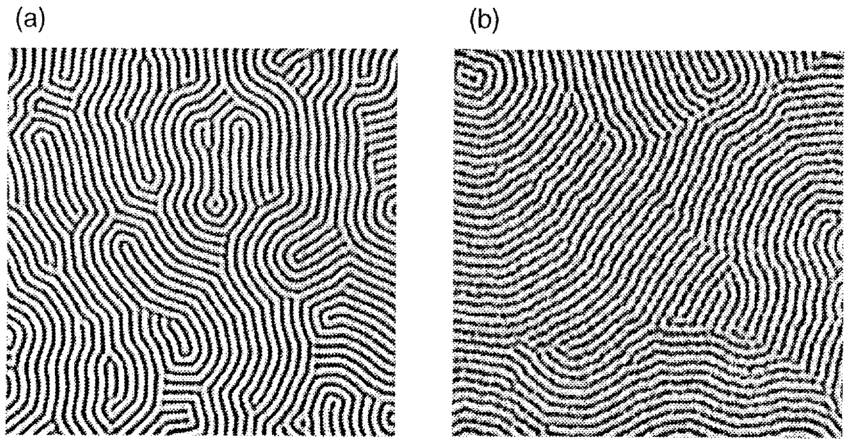

Figure 3. Flow patterns achieved at 10^5 time step for $B = 0$ (a) and 0.06 (b) with the same initial condition. The bright regions denote negative values of the field ψ while the dark ones positive ψ. Each figure exhibits a central 256^2 portion of the 1024^2 lattice result.

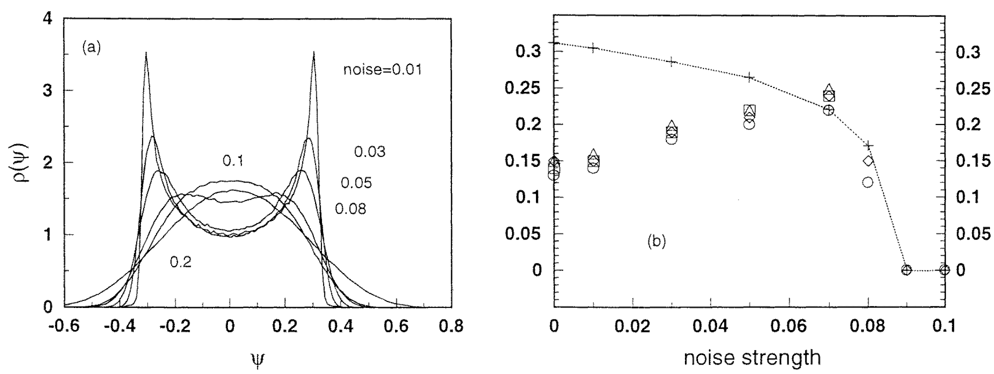

Figure 4. (a) The normalized one-point distribution function $\rho(\psi)$ for various noise intensities (B) at $t = 10^5$. (b) Growth exponent (α) versus noise strength (B). The results plotted were obtained using either $\ell(t)$ (triangles) and $S_p(t)$ (squares) from hardened data or $\ell(t)$ (diamonds) and $S_p(t)$ (circles) from the data without hardening. Also shown by daggers is the position of the peak in $\rho(\psi)$ at $t = 10^5$.

clear that there is a dependence of α on the value of B. Namely, α first becomes progressively larger for higher values of B, and the value of α drops rather abruptly as B increases further. In the same figure, we have also indicated that in the same transition region where the sharp decrease of the exponent occurs, the position of the peak in $\rho(\psi)$ also moves toward $\psi = 0$. We regard our result as being strongly suggestive of a breakdown of the universality hypothesis. At the same time, it clearly describes the enhancement of pattern coarsening at an optimum noise level. The latter characteristic is quite akin to the well-known feature of the stochastic resonance phenomena. (Of course, this analogy is only superficial; the characteristics is due to the existence of a phase transition in our system, not the resonance effect.) In this connection it should be worth adding that, by injection of noise into the time-periodically modulated SH model, Vilar and Rubí [19] have demonstrated the appearance of a maximum in the output (the Nusselt number in this case) signal-to-noise ratio at an optimal dose of noise.

The noised-induced cooperative behavior seems to be at its maximum in the vicinity of the transition from order (with well-defined domains) to disorder. This bears a close resemblance to a notion (known as fitness landscape in biological evolution [20]) of the complex adaptive system. It has been proposed that the system functions at the transition between order and disorder. Deep in the ordered regime, the system would be likely to get stuck because of rigid and conflicting constraints and have no opportunity to reach greater fitness. If it is jiggling around in a random way, that will give a chance to escape from local maxima of fitness and find much higher fitness peaks nearby. Deep in the disordered regime, however, there is too much jiggling and the above strategy will cease to work.

To sum up, we have demonstrated, using the CDS version of SH equation, that the random noise remains relevant in the scaling regime, where the characteristic coarsening length scale exhibits power-law dynamics but the growth exponent depends on the noise level. We believe that this breakdown of universality applies to lamellar ordering in general, and we hope we can report some theoretical analysis of this point in the near future.

YS thanks S Puri for informative conversation.

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